Direct and inverse scattering transform algorithm for complex wave fields

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The discovery of the complete integrability of some nonlinear partial differential equations has stimulated impressive progress in mathematical physics of nonlinear waves. Among these equations are the focusing and defocusing nonlinear Schrodinger (NLSE) equations serving as the fundamental models of nonlinear optics [1]. This breakthrough has taken place due to the development of the Inverse Scattering Transform (IST) method allowing one to solve the initial-value problem in terms of the nonlinear harmonics decomposition representing the scattering data or the IST spectrum [2]. The spectrum can be found using the Direct Scattering Transform (DST), leading to the full knowledge of the nonlinear wave field evolution governed by the integrable differential equation, while the IST procedure allows one to reconstruct the wave field. In some cases, the DST and the IST problems can be solved analytically, but in the general case - which is the subject of this work - only numerically. After several decades of analytical studies of integrable equations, the rapid growth of interest to describe arbitrary shaped, noisy, and even random nonlinear wave fields has promoted the need for accurate numerical methods for the IST/DST. For example, the recent applications of the DST/IST techniques in nonlinear optics studies are ranging from experimental observation of complex interactions of the NLSE breathers [3] and statistically stationary state of spontaneous modulation instability [4] to the development of novel optical telecommunication methods [5,6]. In this work, we take advantage of two efficient numerical techniques: the DST algorithm based on the Magnus expansion [7] and IST algorithm based on the Toeplitz inner bordering scheme [8]. Using a combination of these methods, we demonstrate efficient direct and the inverse scattering transform for various examples of complex optical pulses. Our approach allows to find the whole IST spectrum of an arbitrary wave field and then reproduce each of the spectrum components separately in order to study their nonlinear role in the signal formation. Finally, we discuss the mitigation of the numerical instabilities of the DST and IST in the presents of a large number of solitons in the wave field, see [9], using the application of arbitrary precision arithmetic.

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